

Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Monday 19 May 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2).
There are 11 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Find $\int (8x^3 + 4) dx$, giving each term in its simplest form. (3)

2. (a) Write down the value of $32^{\frac{1}{5}}$. (1)

(b) Simplify fully $(32x^5)^{-\frac{2}{5}}$. (3)

3. Find the set of values of x for which

(a) $3x - 7 > 3 - x$, (2)

(b) $x^2 - 9x \leq 36$, (4)

(c) **both** $3x - 7 > 3 - x$ **and** $x^2 - 9x \leq 36$. (1)

4.

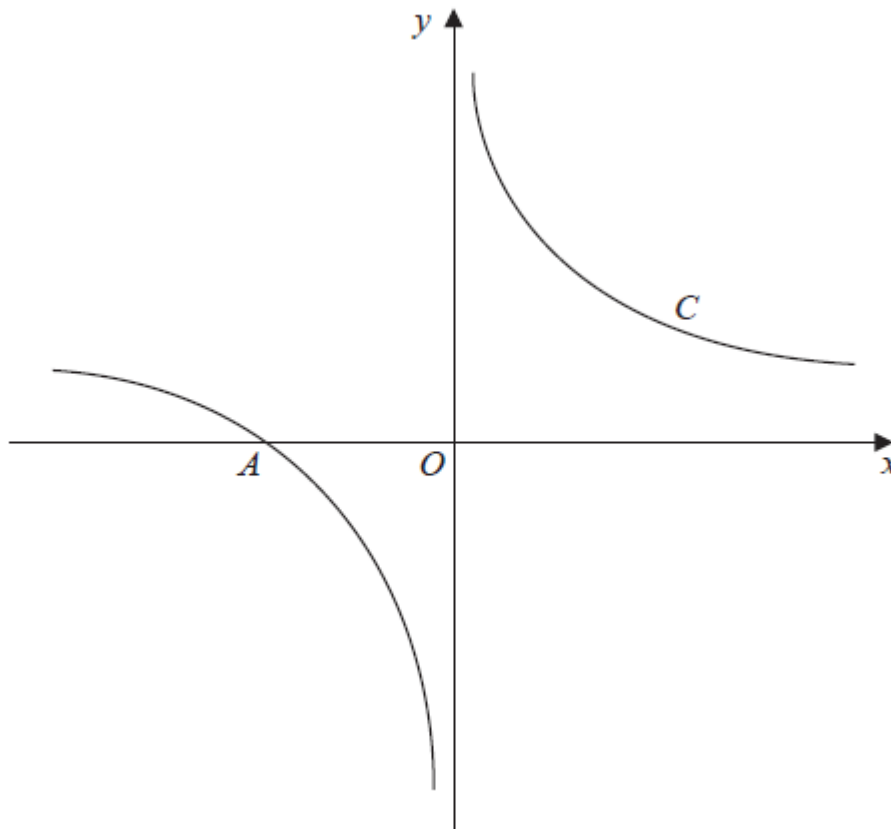


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{1}{x} + 1, \quad x \neq 0.$$

The curve C crosses the x -axis at the point A .

(a) State the x -coordinate of the point A .

(1)

The curve D has equation $y = x^2(x - 2)$, for all real values of x .

(b) On a copy of Figure 1, sketch a graph of curve D . Show the coordinates of each point where the curve D crosses the coordinate axes.

(3)

(c) Using your sketch, state, giving a reason, the number of real solutions to the equation

$$x^2(x - 2) = \frac{1}{x} + 1.$$

(1)

5. A sequence of numbers a_1, a_2, a_3, \dots is defined by

$$a_{n+1} = 5a_n - 3, \quad n \geq 1.$$

Given that $a_2 = 7$,

- (a) find the value of a_1 .

(2)

- (b) Find the value of $\sum_{r=1}^4 a_r$.

(3)

6. (a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant.

(1)

A rectangle R has a length of $(1 + \sqrt{5})$ cm and an area of $\sqrt{80}$ cm².

- (b) Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found.

(4)

7. Differentiate with respect to x , giving each answer in its simplest form,

- (a) $(1 - 2x)^2$,

(3)

- (b) $\frac{x^5 + 6\sqrt{x}}{2x^2}$.

(4)

8. In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(a) Show that the shop sold 220 computers in 2007. (2)

(b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive. (3)

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred. (4)

9.

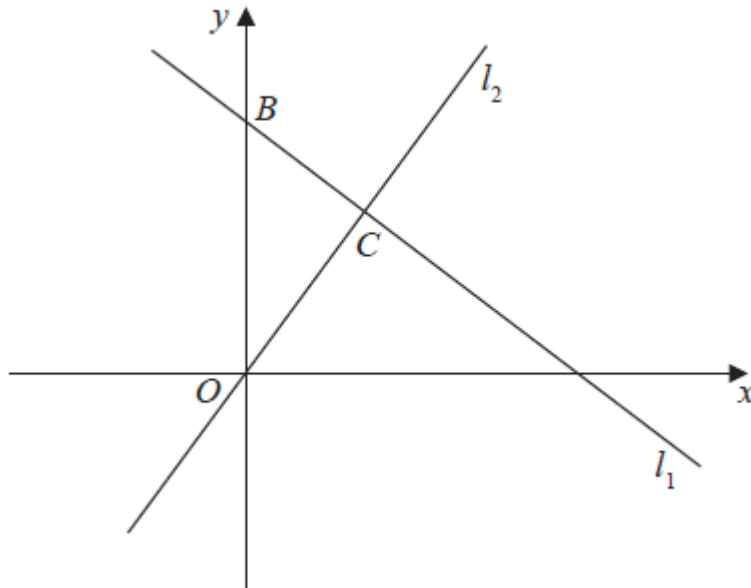


Figure 2

The line l_1 , shown in Figure 2 has equation $2x + 3y = 26$.

The line l_2 passes through the origin O and is perpendicular to l_1 .

(a) Find an equation for the line l_2 .

(4)

The line l_2 intersects the line l_1 at the point C . Line l_1 crosses the y -axis at the point B as shown in Figure 2.

(b) Find the area of triangle OBC . Give your answer in the form $\frac{a}{b}$, where a and b are integers to be determined.

(6)

10. A curve with equation $y = f(x)$ passes through the point $(4, 25)$.

Given that $f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1$, $x > 0$,

(a) find $f(x)$, simplifying each term.

(5)

(b) Find an equation of the normal to the curve at the point $(4, 25)$. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(5)

11. Given that $f(x) = 2x^2 + 8x + 3$,

(a) find the value of the discriminant of $f(x)$.

(2)

(b) Express $f(x)$ in the form $p(x + q)^2 + r$ where p , q and r are integers to be found.

(3)

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = f(x)$.

(c) Calculate the value of c .

(5)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\int (8x^3 + 4) dx = \frac{8x^4}{4} + 4x$ $= 2x^4 + 4x + c$	M1, A1 A1 (3 marks)

Notes

M1 $x^n \rightarrow x^{n+1}$ so $x^3 \rightarrow x^4$ or $4 \rightarrow 4x$ or $4x^1$

A1 This is for either term with coefficient unsimplified (power must be simplified)– so $\frac{8}{4}x^4$ or $4x$
(accept $4x^1$)

A1 Fully correct simplified solution with c i.e. $2x^4 + 4x + c$ [allow $2x^4 + 4x + cx^0$]

If the answer is given as $\int 2x^4 + 4x + c$, with an integral sign – having never been seen as the fully correct simplified answer without an integral sign – then give M1A1A0 but allow anything before the = sign
e.g. $y = 2x^4 + 4x + c$, $f(x) = 2x^4 + 4x + c$, $\int = 2x^4 + 4x + c$, etc....

If this answer is followed by (for example) $x^4 + 2x + k$ then treat this as **isw** (ignore subsequent work)
If they follow it by finding a value for c , also **isw**, provided correct answer with c has been seen and credited

Question Number	Scheme	Marks
2.	<p>(a) $32^{\frac{1}{5}} = 2$</p> <p>(b) For 2^{-2} or $\frac{1}{4}$ or $\left(\frac{1}{2}\right)^2$ or 0.25 as coefficient of x^k, for any value of k including $k = 0$</p> <p>Correct index for x so Ax^{-2} or $\frac{A}{x^2}$ o.e. for any value of A</p> <p>$= \frac{1}{4x^2}$ or $0.25x^{-2}$</p>	<p>B1 (1)</p> <p>M1</p> <p>B1</p> <p>A1 cao (3)</p> <p>4 Marks</p>

Notes

(a) B1 Answer 2 must be in part (a) for this mark

(b) Look at their final answer

M1 For 2^{-2} or $\frac{1}{4}$ or $\left(\frac{1}{2}\right)^2$ or 0.25 in their answer as coefficient of x^k for numerical value of k

(including $k = 0$) so final answer $\frac{1}{4}$ is M1 B0 A0

B1 Ax^{-2} or $\frac{A}{x^2}$ or equivalent e.g. $Ax^{-\frac{10}{5}}$ or $Ax^{-\frac{50}{25}}$ i.e. correct power of x seen in final answer

May have a bracket provided it is $(Ax)^{-2}$ or $\left(\frac{A}{x}\right)^2$

A1 $\frac{1}{4x^2}$ or $\frac{1}{4}x^{-2}$ or $0.25x^{-2}$ oe but must be correct power **and** coefficient combined correctly and must not be followed by a different wrong answer.

Poor bracketing: $2x^{-2}$ earns M0 B1 A0 as correct power of x is seen in this solution (They can recover if they follow this with $\frac{1}{4x^2}$ etc)

Special case $(2x)^{-2}$ as a **final** answer and $\left(\frac{1}{2x}\right)^2$ can have M0 B1 A0 if the correct expanded answer is not seen

The correct answer $\frac{1}{4x^2}$ etc. followed by $\left(\frac{1}{2x}\right)^2$ or $(2x)^{-2}$, treat $\frac{1}{4x^2}$ as final answer so M1 B1 A1 isw

But the correct answer $\frac{1}{4x^2}$ etc clearly followed by the wrong $2x^{-2}$ or $4x^{-2}$, gets M1 B1 A0 do not ignore subsequent wrong work here

Question Number	Scheme	Marks
3.	(a) $3x - 7 > 3 - x$ $4x > 10$ $x > 2.5, x > \frac{5}{2}, \frac{5}{2} < x$ o.e.	M1 A1 (2)
	(b) Obtain $x^2 - 9x - 36$ and attempt to solve $x^2 - 9x - 36 = 0$ e.g. $(x - 12)(x + 3) = 0$ so $x = 12, -3$ $-3 \leq x \leq 12$	M1 A1 M1A1 (4)
	(c) $2.5 < x \leq 12$	A1cso (1)
		(7 marks)

Notes

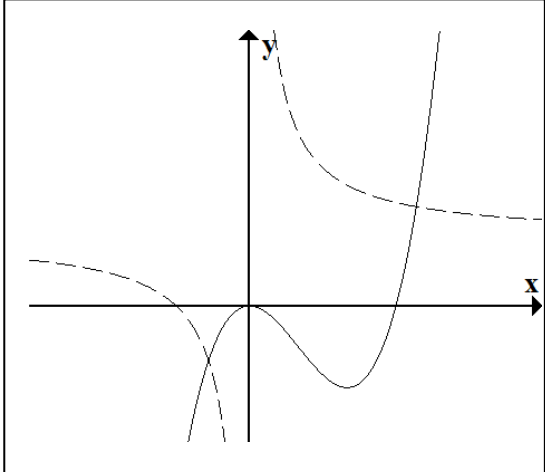
- (a) M1 Reaching $px > q$ with one or both of p or q correct. Also give for $-4x < -10$
A1 Cao $x > 2.5$ o.e. Accept alternatives to 2.5 like $2\frac{1}{2}$ and $\frac{5}{2}$ even allow $\frac{10}{4}$ and allow $\frac{5}{2} < x$ o.e. This answer must occur and be credited as part (a) A correct answer implies M1A1

Mark parts (b) and (c) together.

- (b) M1 Rearrange $3TQ \leq 0$ or $3TQ = 0$ or even $3TQ > 0$ Do not worry about the inequality at this stage AND attempt to solve by factorising, formula or completion of the square with the usual rules (see notes)
A1 12 and -3 seen as critical values
M1 Inside region for their critical values – must be stated – not just a table or a graph
A1 $-3 \leq x \leq 12$ Accept $x \geq -3$ **and** $x \leq 12$ or $[-3, 12]$
For the A mark: Do not accept $x \geq -3$ **or** $x \leq 12$ nor $-3 < x < 12$ nor $(-3, 12)$ nor $x \geq -3, x \leq 12$
However allow recovery if they follow these statements by a correct statement, either in (b) or as they start part (c)
N.B. $-3 \leq 0 \leq 12$ and $x \geq -3, x \leq 12$ are poor notation and get M1A0 here.

- (c) A1 cso $2.5 < x \leq 12$ Accept $x > 2.5$ and $x \leq 12$ Allow $\frac{10}{4}$ Do not accept $x > 2.5$ **or** $x \leq 12$
Accept $(2.5, 12]$ A graph or table is not sufficient. **Must follow correct earlier work** – except for special case

Special case (c) $x > 2.5, x \leq 12$; $2.5 < 0 \leq 12$ are poor notation – but if this poor notation has been penalised in (b) then allow A1 here. Any other errors are penalised in both (b) and (c).

Question Number	Scheme	Marks
4.	<p>(a) - 1 accept $(-1, 0)$</p> <p>(b)</p> <div style="text-align: center;">  </div> <p>(c) 2 solutions as curves cross twice</p>	<p>B1 (1)</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p> <p>B1 ft (1)</p> <p>(5 marks)</p>

Notes

N.B. Check original diagram as answer may appear there.

- (a) B1 The x coordinate of A is -1 . Accept -1 or $(-1,0)$ on the diagram or stated with or without diagram
Allow $(0, -1)$ on the diagram if it is on the correct axis.
- (b) *If no graph is drawn then no marks are available in part (b)*
- B1 Correct shape. The position is not important for this mark but the curve must have two clear turning points and be a +ve x^3 curve (with a maximum and minimum)
- B1 The graph touches the origin. Accept touching as a maximum or minimum. There must be a sketch for this mark but sketch may be wrong and this mark is independent of previous mark. Origin is where axes cross and may not be labelled. This may be a quadratic or quartic curve for this mark.
- B1 The graph crosses the x -axis at the point $(2,0)$ **only**. If it crosses at $(2,0)$ and $(0,0)$ this is B0. Accept $(0,2)$ or 2 marked on the correct axis. Accept $(2, 0)$ in the text of the answer provided that the curve crosses the positive x axis. There must be a sketch for this mark. Do not give credit if $(2,0)$ appears only in a table with no indication that this is the intersection point. (If in doubt send to review) Graph takes precedence over text for third B mark.
- (c) B1ft Two (solutions) as **there are two intersections (of the curves)** N.B. Just states 2 with no reason is B0
If the answer states 2 roots and two intersections – or crosses twice this is enough for B1
BUT B0 If there is any wrong **reason** given – e.g. crosses x axis twice, or crosses asymptote twice
Isw – is not used for this mark so any wrong statement listed to follow a correct statement will result in B0
Allow ft – so if their graph crosses the hyperbola once – allow “one solution as there is one intersection”
And if it crosses three times – allow “three solutions as there are three intersections” or four etc..
If it does not cross at all (e.g.negative cubic) – allow “no solutions as there are no intersections”
However in (c) if they have sketched a curve (even a fully correct one) but not extended it to intersect the hyperbola and they put "no points of intersection so no solutions" then this scores B0.
Accept “lines or curves cross over twice, or touch twice, or meet twice...etc as explanation, but need some form of words)

Question Number	Scheme	Marks
5.	<p>(a) $7 = 5a_1 - 3 \Rightarrow a_1 = ..$ $a_1 = 2$</p> <p>(b) $a_3 = "32"$ and $a_4 = "157"$</p> $\sum_{r=1}^{r=4} a_r = a_1 + a_2 + a_3 + a_4$ $= "2" + "7" + "32" + "157"$ $= 198$	<p>M1 A1 (2)</p> <p>M1</p> <p>dM1</p> <p>A1 (3)</p> <p>(5 marks)</p>

Notes

(a) M1 Writes $7 = 5a_1 - 3$ and attempts to solve leading to an answer for a_1 . If they rearrange wrongly before any substitution this is M0

A1 Cao $a_1 = 2$

Special case: Substitutes $n = 1$ into $5n - 3$ and obtains answer 2. This is fortuitous and gets M0A0 but full marks are available on (b).

(b) M1 Attempts to find either their a_3 or their a_4 using $a_{n+1} = 5a_n - 3$, $a_2 = 7$
Needs clear attempt to use formula or is implied by correct answers or correct follow through of their a_3

dM1 (Depends on previous M mark) Sum of their four adjacent terms from the given sequence.

n.b May be given for $9 + a_3 + a_4$ as they may add $2 + 7$ to give 9

(dM0 for sum of an Arithmetic series)

A1 cao 198

Special case

(a) $a_1 = 32$ is M0 A0

(b) Adds for example $7+32+157 + 782 =$ or $32+157 + 782 + 3907$ is M1 M1 A0

Total mark possible is 2 / 5

(This is not treated as a misread – as it changes the question)

Question Number	Scheme	Marks
6.	<p>(a) $80 = 5 \times 16$ $\sqrt{80} = 4\sqrt{5}$</p> <p>Method 1</p> <p>(b) $\frac{\sqrt{80}}{\sqrt{5}+1}$ or $\frac{c\sqrt{5}}{\sqrt{5}+1}$</p> $= \frac{\sqrt{80}}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} \quad \text{or} \quad \frac{\sqrt{80}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$ $= \frac{20-4\sqrt{5}}{4} \quad \text{or} \quad \frac{4\sqrt{5}-20}{-4}$ $= 5-\sqrt{5}$	<p>B1 (1)</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>A1cao</p> <p>(4)</p> <p>(5 marks)</p>

Notes

(a) B1 Accept $4\sqrt{5}$ or $c = 4$ – no working necessary

(b)

(Method 1)

B1ft Only ft on c See $\frac{\sqrt{80}}{\sqrt{5}+1}$ or $\frac{c\sqrt{5}}{\sqrt{5}+1}$

M1 State intention to multiply by $\sqrt{5}-1$ or $1-\sqrt{5}$ in the numerator **and** the denominator

A1 Obtain denominator of 4 (for $\sqrt{5}-1$) **or** -4 (for $1-\sqrt{5}$) **or** correct simplified numerator of $20-4\sqrt{5}$ or $4(5-\sqrt{5})$ **or** $4\sqrt{5}-20$ or $4(\sqrt{5}-5)$ **So either numerator or denominator must be correct**

A1 Correct answer only. Both **numerator and denominator must have been correct and** division of numerator and denominator by 4 has been performed.

Accept $p=5, q=-1$ or accept $5-\sqrt{5}$ or $-\sqrt{5}+5$ Also accept $5-1\sqrt{5}$

(Method 2)

B1ft Only ft on c $(p+q\sqrt{5})(\sqrt{5}+1)=\sqrt{80}$ or $c\sqrt{5}$

M1 Multiply out the lhs and replace $\sqrt{80}$ by $c\sqrt{5}$

A1 Compare rational and irrational parts to give $p+q=4$, **and** $p+5q=0$

A1 Solve equations to give $p=5, q=-1$

Common error:

$\frac{\sqrt{80}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{4\sqrt{5}-20}{4} = \sqrt{5}-5$ gets B1 M1 A1 (for correct numerator – denominator is wrong for their product) then A0

Correct answer with no working – send to review – have they used a calculator?

Correct answer after trial and improvement with evidence that $(5-\sqrt{5})(\sqrt{5}+1)=\sqrt{80}$ could earn all four marks

Question Number	Scheme	Marks
7.	(a) $(1-2x)^2 = 1-4x+4x^2$ $\frac{d}{dx}(1-2x)^2 = \frac{d}{dx}(1-4x+4x^2) = -4+8x$ o.e.	M1 M1A1 (3)
	Alternative method using chain rule: Answer of $-4(1-2x)$	M1M1A1 (3)
	(b) $\frac{x^5+6\sqrt{x}}{2x^2} = \frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2} = \frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$ Attempts to differentiate $x^{-\frac{3}{2}}$ to give $kx^{-\frac{5}{2}}$ $= \frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$ o.e. Quotient Rule (May rarely appear) – See note below	M1,A1 M1 A1 (4) (7 marks)

Notes

- (a) M1 Attempt to multiply out bracket. Must be 3 or 4 term quadratic and **must have constant term 1**
M1 $x^n \rightarrow x^{n-1}$. Follow through on any term in an incorrect expression. Accept a constant $\rightarrow 0$
A1 $-4+8x$ Accept $-4(1-2x)$ or equivalent. This is not cso and may follow error in the constant term
Following correct answer by $-2+4x$ – apply isw

Correct answer with no working – assume chain rule and give M1M1A1 i.e. 3/3

Common errors: $(1-2x)^2 = 2-4x+4x^2$ is M0, then allow M1A1 for $-4+8x$

$(1-2x)^2 = 1-4x^2$ is M0 then $-8x$ earns M1A0 or $(1-2x)^2 = 1-2x^2$ is M0 then $-4x$ earns M1A0

Use of Chain Rule:

M1M1: first M1 for complete method so $2 \times (\pm 2)(1-2x)$ second M1 for $(1-2x)$ (as power reduced)

Then A1 for $-4(1-2x)$ or for $-4+8x$

So (i) $2(1-2x)$ gets M0 M1A0 for reducing power and (ii) $2 \times 2(1-2x)$ gets M1 M1A0

- (b) M1 An attempt to divide by $2x^2$ first. This can be implied by the sight of the following

Some correct working e.g. $\frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2}$ or $(x^5+6\sqrt{x})(2x^2)^{-1}$ **leading to** $ax^p + bx^q$ in either case

or can be implied by $\frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$ (after no working) i.e. both coefficients correct and power 3 correct

Common error: $(x^5+6\sqrt{x})2x^{-2}$ is M0 (may earn next M mark for the differentiation $x^{-\frac{3}{2}} \rightarrow x^{-\frac{5}{2}}$)

A1 Writing the given expression as $\frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$ or $0.5x^3 + \frac{6}{2}x^{-\frac{3}{2}}$ or $0.5x^3 + \frac{6}{2}x^{-\frac{1}{2}}$ or etc...

M1 $x^{-\frac{3}{2}} \rightarrow x^{-\frac{5}{2}}$ A1 Cao $\frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$ o.e. e.g. $\frac{3}{2}x^2 - \frac{9}{2x^2\sqrt{x}}$ then isw. Allow factorised form. Do not

penalise $+-\frac{9}{2}x^{-\frac{5}{2}}$ used instead of $-\frac{9}{2}x^{-\frac{5}{2}}$

Use of Quotient Rule: M1,A1: Reaching $\frac{2x^2(5x^4+3x^{-\frac{1}{2}}) - 4x(x^5+6x^{\frac{1}{2}})}{4x^4} = \frac{6x^6-18x^{\frac{3}{2}}}{4x^4}$

Send to review if doubtful M1A1: Simplifying (e.g. dividing numerator and denominator by 2) to reach $\frac{3x^6-9x^{\frac{3}{2}}}{2x^4}$ o.e.

Question Number	Scheme	Marks
8.	(a) Use n^{th} term $= a + (n-1)d$ with $d = 10$; $a = 150$ and $n = 8$, or $a = 160$ and $n = 7$, or $a = 170$ and $n = 6$: $= 150 + 7 \times 10$ or $160 + 6 \times 10$ or $170 + 5 \times 10 = 220^*$ (Or gives clear list – see note)	M1 A1* (2)
Or	If answer 220 is assumed and $150 + (n-1)10 = 220$ or variation is solved for $n =$ Then $n = 8$, so 2007 is the year (must conclude the year)	M1 A1* (2)
	(b) Use $S_n = \frac{n}{2}\{2a + (n-1)d\}$ Or $S_n = \frac{n}{2}\{a + l\}$ and $l = a + (n-1)d$ $= 7(300 + 13 \times 10)$ or $7(150 + 280)$ $= 7 \times 430$ $= 3010$	M1 A1 A1 (3)
	(c) Cost in year $n = 900 + (n-1) \times -20$ Sales in year $n = 150 + (n-1) \times 10$ Cost $= 3 \times$ Sales $\Rightarrow 900 + (n-1) \times -20 = 3 \times (150 + (n-1) \times 10)$ $900 - 20n + 20 = 450 + 30n - 30$ $500 = 50n$ $n = 10$ Year is 2009	M1 M1 M1 A1 (4)
	As n is not defined they may work correctly from another base year to get the answer 2009 and their n may not equal 10. If doubtful – send to review.	(9 marks)

Notes

(a) M1 Attempt to use n^{th} term $= a + (n-1)d$ with $d = 10$, and correct combination of a and n i.e. $a = 150$ and $n = 8$ or $a = 160$ and $n = 7$, or $a = 170$ and $n = 6$

A1 * Shows that 220 computers are sold in 2007 with no errors

Note that this is a given solution, so needed $150 + 7 \times 10$ or $160 + 6 \times 10$ or $170 + 5 \times 10$ or equivalent.

Accept a correct list showing all values and years for both marks Just 150,160,170,180,190,200,210,220 is M1A0
Need some reference to years as well as the list of numbers of computers for A1.

(b) M1 Attempts to use $S_n = \frac{n}{2}\{2a + (n-1)d\}$ with $d = 10$, and correct combination of a and n i.e. $a = 150$ and $n = 14$, or $a = 160$ and $n = 13$, or $a = 170$ and $n = 12$

A1 Uses $S_n = \frac{n}{2}\{2a + (n-1)d\}$ with $a = 150$, $d = 10$ and $n = 14$ [N.B. $S_n = \frac{n}{2}\{a + l\}$ needs $l = a + (n-1)d$ as well

NB A0 for $a = 160$ and $n = 13$ or $a = 170$ and $n = 12$ unless they then add the first, or first two terms respectively.

A1 Cao 3010. This answer (with no working) implies correct method M1A1A1.

Special case: If a complete list $150 + 160 + 170 + 180 + 190 + 200 + 210 + 220 + 230 + 240 + 250 + 260 + 270 + 280$ is seen, then there is an error finding the sum then score M1A1A0, but incomplete or wrong lists score M0A0A0

(c) M1 Writes down an expression for the cost $= 900 + (n-1) \times -20$ or writes $900 + (n-1)d$ and states $d = -20$
Allow $900 + n \times -20$. Allow recovery from invisible brackets.

M1 **Attempts** to write down an equation in n for statement ‘cost $= 3 \times$ sales’

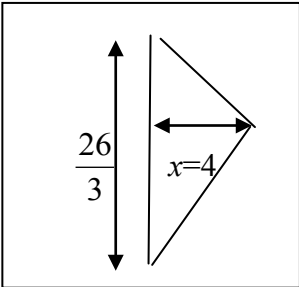
$900 + (n-1) \times -20 = 3 \times (150 + (n-1) \times 10)$. Accept the 3 on the wrong side and allow use of 20 instead of -20 and allow n (consistently) instead of $n - 1$ for this mark. Ignore £ signs in equation.

M1 Solves the correct linear equation in n to achieve $n = 10$ (for those using $n - 1$) or $n = 9$ (for those using n).
Ignore £ signs.

A1 Cso Year 2009 (A0 for the answer Year 10 if 2009 is not given)

Special case. **Just answer or trial and improvement** with no equation leading to answer scores SC 0,0,1,1

Equations satisfying the method mark descriptors followed by trial and improvement could get all four marks

Question Number	Scheme	Marks
9.	<p>(a) $2x + 3y = 26 \Rightarrow 3y = 26 - 2x$ and attempt to find m from $y = mx + c$</p> <p>($\Rightarrow y = \frac{26}{3} - \frac{2}{3}x$) so gradient = $-\frac{2}{3}$</p> <p>Gradient of perpendicular = $\frac{-1}{\text{their gradient}} (= \frac{3}{2})$</p> <p>Line goes through (0,0) so $y = \frac{3}{2}x$</p> <p>(b) Solves their $y = \frac{3}{2}x$ with their $2x + 3y = 26$ to form equation in x or in y</p> <p>Solves their equation in x or in y to obtain $x =$ or $y =$</p> <p>$x=4$ or any equivalent e.g. $156/39$ or $y = 6$ o.a.e</p> <p>$B = (0, \frac{26}{3})$ used or stated in (b)</p> <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 10px; margin-right: 10px;">  </div> <div> <p>Method 1 (see other methods in notes below)</p> <p>Area = $\frac{1}{2} \times "4" \times \frac{"26"}{3}$</p> <p>= $\frac{52}{3}$ (oe with integer numerator and denominator)</p> </div> </div>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>B1</p> <p>dM1</p> <p>A1</p> <p>(6)</p> <p>(10 marks)</p>

Notes

(a) M1 Complete method for finding gradient. (This may be implied by later correct answers.)
e.g. Rearranges $2x + 3y = 26 \Rightarrow y = mx + c$ so $m =$

Or finds coordinates of two points on line and finds gradient e.g. (13,0) and (1,8) so $m = \frac{8-0}{1-13}$

A1 States or implies that gradient = $-\frac{2}{3}$ - condone $-\frac{2}{3}x$ if they continue correctly. Ignore errors in constant term in straight line equation

M1 Uses $m_1 \times m_2 = -1$ to find the gradient of l_2 . This can be implied by the use of $\frac{-1}{\text{their gradient}}$

A1 $y = \frac{3}{2}x$ or $2y - 3x = 0$ Allow $y = \frac{3}{2}x + 0$ Also accept $2y=3x, y=39/26x$ or even $y - 0 = \frac{3}{2}(x - 0)$ and isw

Notes Continued

- (b) M1 Eliminates variable between their $y = \frac{3}{2}x$ and their (possibly rearranged) $2x + 3y = 26$ to form an equation in x or y . (They may have made errors in their rearrangement)
- dM1 (Depends on previous M mark) Attempts to solve their equation to find the value of x or y
- A1 $x = 4$ or equivalent or $y = 6$ or equivalent
- B1 y coordinate of B is $\frac{26}{3}$ (stated or implied) - isw if written as $(\frac{26}{3}, 0)$. **Must be used or stated in (b)**
- dM1 (Depends on previous M mark) Complete method to find area of triangle OBC (using their values of x and/or y at point C and their $26/3$)
- A1 Cao $\frac{52}{3}$ or $\frac{104}{6}$ or $\frac{1352}{78}$ o.e

Method 1:

Uses the area of a triangle formula $\frac{1}{2} \times OB \times (x \text{ coordinate of } C)$

Alternative methods: Several Methods are shown below. The only mark which differs from Method 1 is the last M mark and its use in each case is described below:

Method 2 in 9(b) using $\frac{1}{2} \times BC \times OC$

dM1 Uses the area of a triangle formula $\frac{1}{2} \times BC \times OC$ Also finds $OC (= \sqrt{52})$ and $BC = (\frac{4}{3}\sqrt{13})$

Method 3 in 9(b) using $\frac{1}{2} \begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 6 & \frac{26}{3} & 0 \end{vmatrix}$

dM1 States the area of a triangle formula $\frac{1}{2} \begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 6 & \frac{26}{3} & 0 \end{vmatrix}$ or equivalent with their values

Method 4 in 9(b) using area of triangle OBX – area of triangle OCX where X is point $(13, 0)$

dM1 Uses the correct subtraction $\frac{1}{2} \times 13 \times \frac{26}{3} - \frac{1}{2} \times 13 \times 6$

Method 5 in 9(b) using area = $\frac{1}{2} (6 \times 4) + \frac{1}{2} (4 \times 8/3)$ drawing a line from C parallel to the x axis and dividing triangle into two right angled triangles

dM1 for correct method area = $\frac{1}{2} ("6" \times "4") + \frac{1}{2} ("4" \times ["26/3" - "6"])$

Method 6 Uses calculus

dM1 $\int_0^4 \left(\frac{26}{3} - \frac{2x}{3} - \frac{3x}{2} \right) dx = \left[\frac{26}{3}x - \frac{x^2}{3} - \frac{3x^2}{4} \right]_0^4$

Question Number	Scheme	Marks
10.	<p>(a) $f(x) = \int \left(\frac{3}{8}x^2 - 10x^{\frac{1}{2}} + 1 \right) dx$</p> $x^n \rightarrow x^{n+1} \Rightarrow f(x) = \frac{3}{8} \times \frac{x^3}{3} - 10 \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}} + x(+c)$ <p>Substitute $x = 4, y = 25 \Rightarrow 25 = 8 - 40 + 4 + c \Rightarrow c =$</p> $(f(x)) = \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53$ <p>(b) Sub $x=4$ into $f'(x) = \frac{3}{8}x^2 - 10x^{\frac{1}{2}} + 1$</p> $\Rightarrow f'(4) = \frac{3}{8} \times 4^2 - 10 \times 4^{\frac{1}{2}} + 1$ $\Rightarrow f'(4) = 2$ <p>Gradient of tangent = 2 \Rightarrow Gradient of normal is $-1/2$</p> <p>Substitute $x = 4, y = 25$ into line equation with their changed gradient</p> <p>e.g. $y - 25 = -\frac{1}{2}(x - 4)$</p> $\pm k(2y + x - 54) = 0 \quad \text{o.e. (but must have integer coefficients)}$	<p>M1, A1, A1</p> <p>M1</p> <p>A1</p> <p>(5)</p> <p>M1</p> <p>A1</p> <p>dM1</p> <p>dM1</p> <p>A1cso</p> <p>(5)</p> <p>(10 Marks)</p>

Notes

- (a) M1 Attempt to integrate $x^n \rightarrow x^{n+1}$
- A1 Term in x^3 **or** term in $x^{\frac{1}{2}}$ correct, coefficient need not be simplified, no need for $+x$ nor $+c$
- A1 ALL three terms correct, coefficients need not be simplified, no need for $+c$
- M1 For using $x = 4, y = 25$ in their $f(x)$ to form a linear equation in c and attempt to find c
- A1 $= \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53$ cao (all coefficients and powers must be simplified to give this answer- do not need a left hand side and if there is one it may be $f(x)$ or y). Need full expression with 53
These marks need to be scored in part (a)
- (b) M1 Attempt to substitute $x = 4$ into $f'(x)$ must be in part (b)
- A1 $f'(x) = 2$ at $x = 4$
- dM1 (Dependent on first method mark in part (b)) Using $m_1 \times m_2 = -1$ to find the gradient of the normal from their tangent gradient (Give mark if gradient of 1 becomes -1 as they will lose accuracy)
- dM1 (Dependent on first method mark in part (b)) Attempt to find the equation of the normal (not tangent). Eg use $x=4, y=25$ in $y = -1/2x + c$ to find a value of c or use $-\frac{1}{2} = \frac{y-25}{x-4}$ with their adapted gradient.
- A1 cso $\pm k(2y + x - 54) = 0$ (where k is any integer)

Question Number	Scheme	Marks
11.	(a) Discriminant = $b^2 - 4ac = 8^2 - 4 \times 2 \times 3 = 40$	M1, A1 (2)
	(b) $2x^2 + 8x + 3 = 2(x^2 + \dots)$ or $p=2$ $= 2((x+2)^2 \pm \dots)$ or $q=2$ $= 2(x+2)^2 - 5$ or $p=2, q=2$ and $r=-5$	B1 M1 A1 (3)
	(c) Method 1A: Sets derivative " $4x+8$ " = $4 \Rightarrow x = , x = -1$ Substitute $x = -1$ in $y = 2x^2 + 8x + 3 \Rightarrow y = -3$ Substitute $x = -1$ and $y = -3$ in $y = 4x + c$ or into $(y+3) = 4(x+1)$ and expand $c = 1$ or writing $y = 4x + 1$	M1, A1 dM1 dM1 A1cso (5)
	Method 1B: Sets derivative " $4x+8$ " = $4 \Rightarrow x = , x = -1$ Substitute $x = -1$ in $2x^2 + 8x + 3 = 4x + c$ Attempts to find value of c $c = 1$ or writing $y = 4x + 1$	M1, A1 dM1 dM1 A1cso (5)
	Method 2: Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent States that $b^2 - 4ac = 0$ $4^2 - 4 \times 2 \times (3 - c) = 0$ and so $c =$ $c = 1$	M1 A1 dM1 dM1 A1cso (5)
	Method 3: Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent Uses $2(x+1)^2 - 2 + 3 - c = 0$ or equivalent Writes $-2 + 3 - c = 0$ So $c = 1$	M1 A1 dM1 dM1 A1cso (5)
	Also see special case for using a perpendicular gradient (overleaf)	(10 marks)

Notes

- (a) M1 Attempts to calculate $b^2 - 4ac$ using $8^2 - 4 \times 2 \times 3$ - must be correct – not just part of a quadratic formula
A1 Cao 40
- (b) B1 See $2(\dots)$ or $p=2$
M1 $\dots((x+2)^2 \pm \dots)$ is sufficient evidence or obtaining $q=2$
A1 Fully correct values. $2(x+2)^2 - 5$ or $p=2, q=2, r=-5$ cso.
Ignore inclusion of “=0”.

[In many respects these marks are similar to three B marks.

$p=2$ is B1; $q=2$ is B1 and $p=2, q=2$ and $r=-5$ is final B1 but they must be entered on open as **B1 M1 A1**]

Special case: Obtains $2x^2 + 8x + 3 = 2(x+2) - 1$ This may have first B1, for $p=2$ then M0A0

(c) Method 1A (Differentiates and puts gradient equal to 4. Needs both x and y to find c)

M1 Attempts to solve their $\frac{dy}{dx} = 4$. They must reach $x = \dots$ (Just differentiating is M0 A0)

A1 $x = -1$ (If this follows $\frac{dy}{dx} = 4x + 8$, then give M1 A1 by implication)

dM1 (Depends on previous M mark) Substitutes **their** $x = -1$ into $f(x)$ or into “their $f(x)$ from (b)” to find y

dM1 (Depends on both previous M marks) Substitutes **their** $x = -1$ and **their** $y = -3$ values into $y = 4x + c$ to find c or uses equation of line is $(y + “3”) = 4(x + “1”)$ and rearranges to $y = mx + c$

A1 $c = 1$ or allow for $y = 4x + 1$ cso

(c) Method 1B (Differentiates and puts gradient equal to 4. Also equates equations and uses x to find c)

M1A1 Exactly as in Method 1A above

dM1 (Depends on previous M mark) Substitutes **their** $x = -1$ into $2x^2 + 8x + 3 = 4x + c$

dM1 Attempts to find value of c then A1 as before

(c) Method 2 (uses repeated root to find c by discriminant)

M1 Sets $2x^2 + 8x + 3 = 4x + c$ and tries to collect x terms together

A1 Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$ or even $2x^2 + 4x = c - 3$ Allow “=0” to be missing on RHS.

dM1 (If the line is a tangent it meets the curve at just one point so repeated root and $b^2 - 4ac = 0$)
Stating that $b^2 - 4ac = 0$ is enough

dM1 Using $b^2 - 4ac = 0$ to obtain equation in terms of c

(Eg. $4^2 - 4 \times 2 \times (3 - c) = 0$) AND leading to a solution for c

A1 $c = 1$ or allow for $y = 4x + 1$ cso

(c) Method 3 (Similar to method 2 but uses completion of the square on the quadratic to find repeated root)

M1 Sets $2x^2 + 8x + 3 = 4x + c$ and tries to collect x terms together. May be implied by $2x^2 + 8x + 3 - 4x \pm c$ on one side

A1 Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$ or even $2x^2 + 4x = c - 3$ Allow “=0” to be missing on RHS.

dM1 Then use completion of square $2(x+1)^2 - 2 + 3 - c = 0$ (Allow $2(x+1)^2 - k + 3 - c = 0$) where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square

dM1 $-2 + 3 - c = 0$ AND leading to a solution for c (Allow $-1 + 3 - c = 0$) ($x = -1$ has been used)

A1 $c = 1$ cso

In Method 1 they may use part (b) and differentiate their $f(x)$ and put it equal to 4

They can earn M1, but do not follow through errors.

In Methods 2 and 3 they may use part (b) to write

their $2(x+2)^2 - 5 = 4x + c$. They need to expand and collect x terms together for M1

Then expanding gives $2x^2 + 4x + 3 - c = 0$ for A1 – do not follow through errors

Then the scheme is as before

If they just state $c = 1$ with little or no working – please send to review,

PTO for special case

Special case uses perpendicular gradient (maximum of 2/5)

Sets $4x + 8 = -\frac{1}{4} \Rightarrow x = , \quad x = -\frac{33}{16}$ M1 A0

Substitute $x = -\frac{33}{16}$ in $y = 2x^2 + 8x + 3$ ($\Rightarrow y = -\frac{639}{128}$) M0

Substitute $x = -\frac{33}{16}$ and $y = -\frac{639}{128}$ into $y = 4x + c$ or into $(y + \frac{639}{128}) = 4(x + \frac{33}{16})$ and expand M1 A0